

Strain measurement using EBSD: An introduction without the math. By Dr. David Dingley

In the examples below I have used BLG Productions Crosscourt 3 software and would like to thank Dr Graham Meaden for it.

Many materials scientists feel unsure of the new strain measurement application of EBSD. If you read the papers and are not familiar with tensors and in particular the strain tensor, the application can look a bit daunting. There is also some current controversy over absolute versus relative strain measurement and what can and cannot be done. This cannot be good for those new into the field. There needs to be some simple guidance through the maze. This article sets out some easy ideas to get you started.

Perhaps you are asking the question what is special about measuring strain using EBSD. Well for a start it provides a full description of the residual strain within a sample. That is it yields full data on the elastic strain components as well as a partial measure of the dislocation density after plastic deformation. Furthermore, it provides the data at a spatial resolution of the order 60nm. We can compare this with other techniques for measuring residual strain as in the figure shown below.

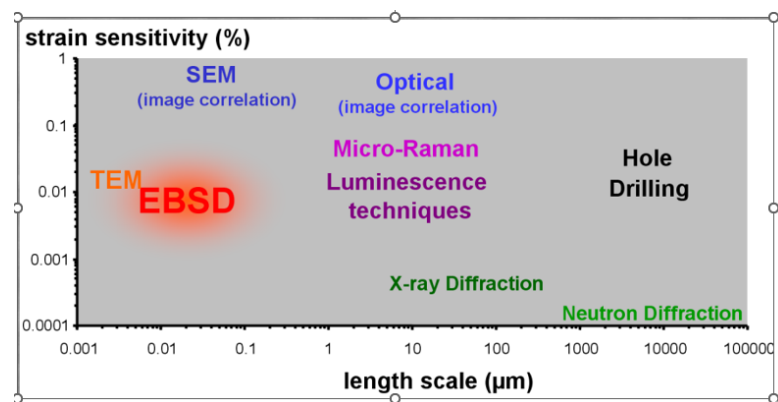


Image Courtesy of Angus Wilkinson (Oxford University)

I add the comment that only the TEM based technique of Convergent Beam Diffraction can match what is possible using the EBSD strain tool as described below. Kossel x-ray diffraction is the only x-ray technique that can provide a full description of residual strain; other x ray techniques can only provide partial descriptions. Though the Kossel technique provides a higher precision it has a spatial resolution 100 times poorer.

Well we all know of situations where we would like to know the internal strain (or stress) state for example.

- Are the strains ahead of a crack compressive or tensile?
- Are there local shear forces present, those that cause atom planes to slide over each other, which are of sufficient magnitude such that dislocation sources might be activated?
- What is going on at a grain boundary triple junction?

Just asking these questions actually shows that you know quite a bit about strains already. You probably know there are two sorts of strains (with their associated stresses), tensile and shear. You also know about crystal rotation and you know that small rotations can occur at grain boundaries even in well annealed material. You would probably not be reading this if you had no experience of mechanical testing in which case you know that the strains we are talking about are those that occur at the beginning of a tensile test where the stress strain curve is linear, that is the elastic strains, and you know these are very small. You are also aware of the strict definition of elastic strain which is that strain which is completely recoverable on removing the constraint (load).

The yield point in most metals for example is at about 0.1% strain. In the literature on strain measurement we refer to fractional strain rather than percentage so 0.1% strain is equivalent to 0.001 or 1 part in 1000. You also probably know that these elastic strains persist into the plastic region of deformation and can be magnified by dislocation pile ups etc. What you are probably unfamiliar with is; how are the tensile and shear strains related and how do we express the combined set of strains in a crystal in a formal way. We begin at the very beginning.

Some fundamentals

Hooke's Law and what follows. Well we learnt about this at school. If you hang a weight on a spring it extends in direct proportion to the load. And of course that is the same thing as is going on in the elastic deformation part of the standard tensile test.

We normally write $\text{applied stress} = \text{Young's modulus} * \text{strain}$

That is $\sigma = Y^* e$

Where stress is a force (a vector property) divided by the cross sectional area. Strain is the displacement (also a vector quantity) per unit length and is also a vector quantity. A vector quantity requires three numbers to define its direction. It can be represented then by a number with one subscript, U_i , where i takes the numbers 1 to 3. Such a quantity can be referred to as a tensor of rank 1. The rank refers to the number of subscripts. The strains e can be formally written as a differential or gradient. So that, if u_1 was the extension in the x direction, we would write the strain as $\delta u_1 / \delta x$.

We also know that when we stretch something it gets thinner in the direction normal to the stretch direction. This is the Poisson effect and the ratio extension /contraction is the Poisson ratio.

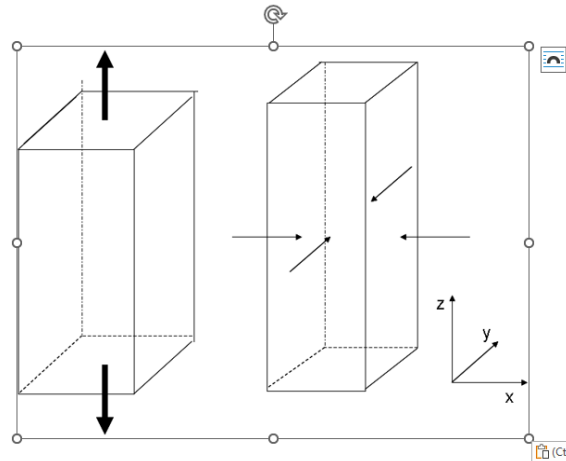


Illustration of Poisson effect.

This coupling of the different strains caused in a sample by the application of a stress classifies strain as a tensor property. In fact it is a second rank tensor because as we shall see it will consist of a 3×3 matrix and hence each element will need two suffixes to locate any one of its components in the matrix, i.e. ϵ_{ij} . The physical significance of this will be apparent later.

Let's set up a co-ordinate system based on some reference axes in the sample. A convenient one for the case illustrated above is included in the figure. It is a right handed system when reading the axes in the sequence x y z . That is if we start at a point on the positive x axis and rotate towards the positive y axis in a clockwise direction then positive z axis would point in the direction of movement of a right handed screw. This is an important point and all data output in the currently only available system to provide the full strain tensor, XC EBSD, (Cross correlated EBSD) and its associated software Crosscourt 3, conforms to this convention.

To keep it as clear as possible which reference axes we are using it is conventional often to refer to these axes as 1 2 and 3. This is to allow easy identification of the strain terms as depicted in the tensor 3×3 matrix. The directions 1 2 and 3 are synonymous with the x y and z directions.

In a sample strained in tension (uniaxial tension) as in the above figure, the z axis has been strained in the positive sense, i.e. it becomes longer. The strain term for this deformation is ϵ_{33} . The applied force is acting in the z direction. The first subscript refers to the axis that is being strained and the second to the direction it is being strained. Thus the first subscript 3 refers to the strain is occurring along the z axis and the second subscript to the fact that the strain is in the direction of the z axis. Likewise for the strains ϵ_{11} and ϵ_{22} . The strains ϵ_{11} , ϵ_{22} and ϵ_{33} are known as tensile or normal strains.

The force along z can also be resolved onto inclined planes causing these planes to shear resulting in shear strains. It will also cause the crystal to rotate (called a rigid body rotation). To illustrate this I include a more general deformation, as shown next.

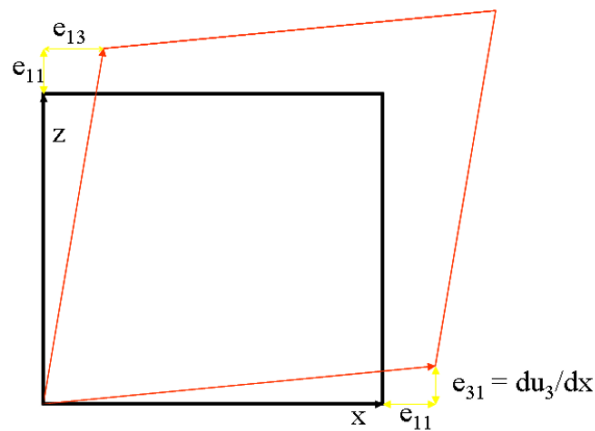


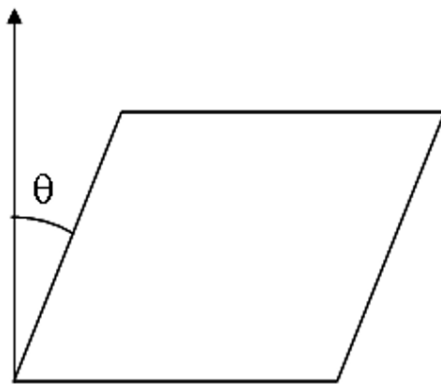
Illustration of tensile and shear strains plus rotation in x z plane.

x y z right handed axis system, u_1 u_2 u_3 displacements in x y z directions.

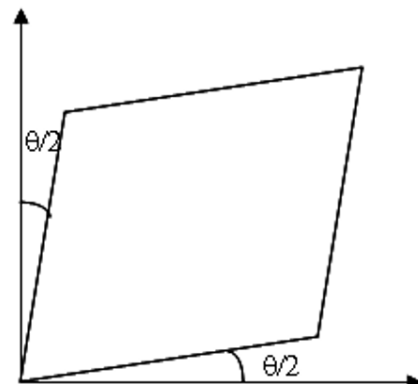
The figure shows only deformation, that is change of length and direction, in the x z plane for clarity. The tensile strains e_{11} and e_{33} are shown plus the two strains e_{13} and e_{31} . Note the differential form of these latter strains as shown in the figure. Let us consider for now that there is no rotation. Then e_{13} and e_{31} represent the shear components alone and are in fact equal. The first of the subscript in both of e_{13} and e_{31} signifies the plane that is sheared and the second component the axis defining the direction of shear. Hence the first shear component e_{13} describes the shear of the plane normal to the x axis and in the z direction whilst the second, e_{31} defines the shear of the plane normal to the z axis in the x direction.

Simple (mechanical) shear versus physics

Now some of you may say, 'surely shear of a plane distorts a crystal' as shown in the left sketch below.



Simple shear. (Engineering)



Shear (Physics)

Well this is true and is how engineers express it. As shown in the right hand figure, the shear strain as used in engineering is twice that used by physicists. Engineering strain measures the total strain in the xz plane and is often given the symbol γ . On the other hand ϵ_{31} , which remember $= \delta u_3 / \delta x$ and in this case where there is no rotation, is simply the average of the strains on the z and x faces i.e. $\epsilon_{31} = \frac{1}{2}(\epsilon_{31} + \epsilon_{13})$. Note I have changed the symbol from ϵ_{31} to ϵ_{31} . The symbol ϵ_{31} refers to the strains as measured and may contain a rotational component. The symbol ϵ_{31} refers specifically to the shear strain component included in the ϵ_{31} term. You may need to note this when involved in discussion with engineers. For consistency the Greek symbol ϵ is used instead of e for the tensile strains as well.

Tensors in Strain

Tensors: Geometric entities introduced into mathematics and physics to extend the notion of scalars, (geometric) vectors, and matrices. Many physical quantities are naturally regarded not as vectors themselves, but as correspondences between one set of vectors and another.

Strain Tensor

Following on from the previous section, in total there are 9 strain components three are the normal strains (tensile or compressive), ϵ_{11} ϵ_{22} ϵ_{33} and six shear strains ϵ_{12} ϵ_{13} ϵ_{21} ϵ_{23} ϵ_{31} ϵ_{32} . These are often written as the matrix.

$$\begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{bmatrix}$$

This matrix is known as the **strain tensor**. We will see that this is necessarily a symmetric matrix in the sense that $\epsilon_{12} = \epsilon_{21}$, $\epsilon_{13} = \epsilon_{31}$, $\epsilon_{23} = \epsilon_{32}$.

Thus if you want to know what is the strain state of a selected region of your sample you can express it completely if you know these 9 (actually 6 as there are three equal pairs) components. There is no more to it than that.

Well actually, in calculating these components from an EBSD pattern there is quite a lot more to it, but it is all dealt with in the CrossCourt 3 software so you don't have to worry about the details, at least as a beginner. (If requested I will prepare a fuller tutorial on how we go from the EBSD pattern to the strain tensor but that is for a later time. Those who can't wait are referred to the several papers by Wilkinson et al on the subject, see bibliography below).

Stress Tensor

Before we proceed to describe what the CrossCourt program does for us we just want to point out what is probably obvious and that is that just as the components of strain are coupled together so too are the associated stresses. Thus there are 9 components of stress and like strain the off axis components are equal. So we have as the **stress tensor** with $\sigma_{12} = \sigma_{21}$, $\sigma_{13} = \sigma_{31}$, $\sigma_{23} = \sigma_{32}$. If these stresses were not respectively equal then the crystal would rotate until they are. The stress tensor is generally written

$$\begin{vmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{vmatrix}$$

Elastic Modulus Tensor

The stress and strain tensors are linked through the elastic modulus tensor. We write for the stress tensor σ_{ij} and for the strain tensor ϵ_{kl} . They are related according to

$$\sigma_{ij} = C_{ijkl} \cdot \epsilon_{kl}$$

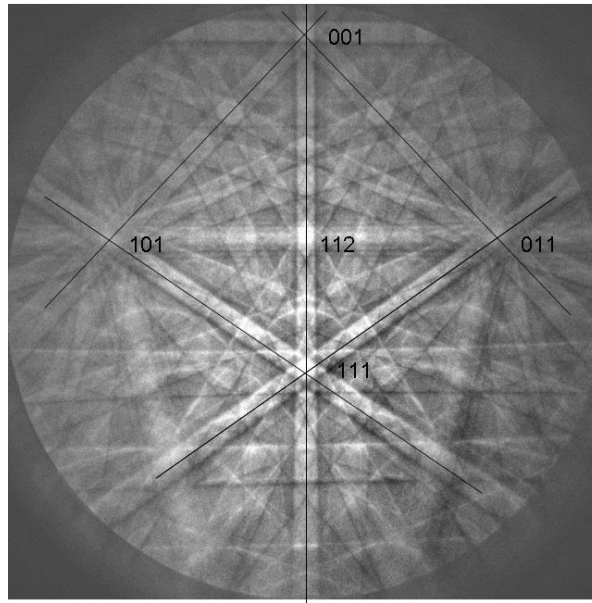
This complete form of Hooke's law is more difficult to handle than the simpler version. The elastic modulus tensor (a fourth rank tensor because it needs 4 subscripts to locate each component) has 81 components. Fortunately crystal symmetry reduces this drastically so that for cubic crystals for example there are only 3 independent components.

Again the Crosscourt user need not concern themselves here with the details of solving for stresses given the strains or vice versa, the software takes care of it. For those now sufficiently excited about the subject I refer them to Prof John Nye's excellent book *Physical Properties of Crystals*, Oxford University Press, 1957.

I will myself post a more extended tutorial depending on requests.

Crystal strain and the EBSD pattern

If the crystal becomes distorted because of residual strain then it follows the EBSD pattern will become distorted and the one can be directly related to the other. In fact the correspondence is quite simple to see. Let's take the uniaxial strain as depicted in the first figure. Then (neglecting for the time being any complications due to grain-grain continuity interactions and any relaxations at the free surfaces) all diffraction patterns obtained from any of the crystals in the sample will show the same uniaxial strain with respect to the sample reference axes. (Of course, the whole point of a micro strain detection system is that we can use the technique to study; anisotropy, local grain boundary strains and the subtle effects of crystal orientation on the strain tensor. However, for now, let's consider homogenous deformation independent of crystal orientation). Suppose we had a crystal in the sample that was in such a particular orientation that a symmetry axis was parallel to the tensile axis and produced the EBSD pattern seen below.



EBSD pattern from cubic crystal oriented such that the 110 band is vertical

A tensile strain in the vertical direction would cause the zone axis $[001]$ to move upward and apart from the $[111]$ axis. The consequent contraction of the sample due to the Poisson effect would cause the axes $[101]$ and $[011]$ to move towards the vertical band. But the movements would be very small. The angle between $[001]$ and $[111]$ is 54.67° . If the strain was to be 0.001 then this angle would change to 54.70° . Thus if we wanted to measure the strain to 1 part in 1000 we need to measure a change in angle between these two zone axes of just 0.03 degrees. The original image contained 1000 pixels top to bottom. It works out that a change of 0.03° would shift the $[111]$ axis only 0.26 pixels away from the $[001]$ axis. For those who have read my comments on strain measurement some 15 years ago I argue that as it was impossible to resolve the image better than one pixel then the measurement of such small strains would be impossible. However, I was premature in my negative appraisal. My assumption that we could not resolve an image shift better than an individual pixel was wrong. In fact using cross correlation methods where two slightly different images are compared, then the small changes in intensity between corresponding pixels in the two images can be analyzed statistically to give an average displacement between the two images which is very much less than one pixel. Currently, using a $1k \times 1k$ image, the sensitivity is $1/20^{\text{th}}$ of a pixel.

In practice, therefore, to measure strain using EBSD patterns we compare two patterns, one from a strained region of the crystal and one from an unstrained region. (We will discuss later the selection of the unstrained region). We then select small regions of the pattern to be compared. We could take patterns at the top and bottom to determine distortion in the vertical sense of the pattern and patterns either side to determine the lateral distortion. In practice we take around twenty regions at points on a circle centered about the image centre. We then compare each region and measure the movement of the

pattern in each case. We then analyze the movements to determine all of the components of the strain tensor.

I am not going into the mathematics of how we do that here but the interested reader will find it fully described in the papers by Wilkinson et al listed in the bibliography at the end of this tutorial. However I present next a synopsis of the procedure.

Synopsis of strain measurement from EBSD patterns

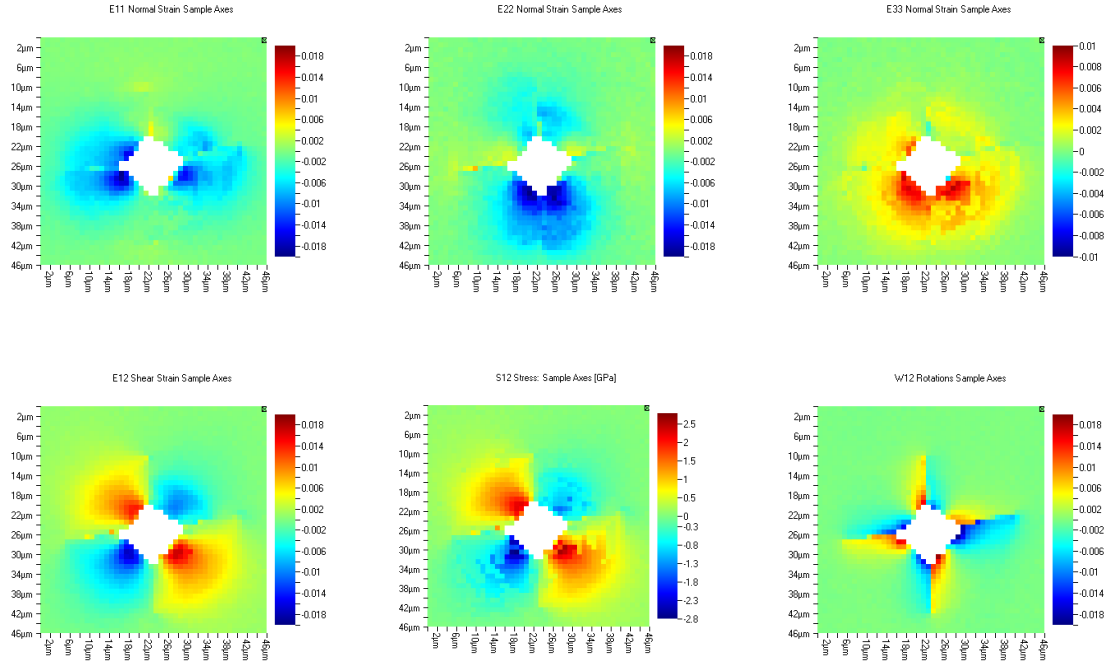
1. The basis of the technique is to record two EBSD patterns one recorded from the strained region and one to serve as a reference pattern. The reference area should be close to the strained area within say 100nm and of similar orientation, up to 7 degrees disorientation is acceptable. The reference area ideally should be of zero strain. (More about this in a second tutorial).
2. What are measured are the displacements between corresponding selected regions (regions of Interest-ROI) in the patterns. At least 4 ROI are needed but many more improve both precision and accuracy.
3. The displacements are interpreted to provide the strain.
4. The first analysis provides 8 of the strain components. We can not determine the ninth component as it is not possible to measure the crystal distortion normal to the phosphor screen on which the patterns are recorded. However it is known that the surface viewed in the SEM and from which the EBSD is recorded is in a traction free condition. That is, no stress acts across it. If it did then it would move until the stress reached zero. We make use of this fact to calculate the strain acting normal to the surface by appropriate substitution into the generalized Hooke's law as given above. (Note, a zero stress does not mean the strain is also zero). There is an implied assumption here though. It is assumed that the stress normal to the free surface remains zero throughout the volume sampled by the electron beam and contributing to the diffraction pattern. The sample depth is thought to be about 20nm so this is not too risky an assumption.
5. The next point is that in the general case there is usually some associated crystal rotation. It is obvious when rotation is present because in the as measured tensor the component e_{12} will not be equal to e_{21} as it should by definition of the shear components. Likewise $e_{13} \neq e_{31}$, $e_{23} \neq e_{32}$. The rotation can be removed from the as measured tensor by splitting the tensor into its symmetric and antisymmetric parts. The symmetric part becomes the true strain tensor and the antisymmetric part a tensor describing the rotation. In EBSD the rotation tensor is normally called the disorientation matrix and in this case describes the disorientation between the unstrained and strained regions.
The symmetric tensor is formed simply by replacing the as measured shear components e_{12} and e_{21} by $\varepsilon_{12} = \varepsilon_{21} = \frac{1}{2}(e_{12} + e_{21})$. Likewise for ε_{13} , ε_{31} , ε_{23} , ε_{32} . The antisymmetric part the tensor describing the rotation is given by the difference between diagonally opposite components of the as measured tensor. Writing ω_{ij} for the off diagonal components of the rotation matrix to distinguish between strain and rotation components we have $\omega_{12} = \frac{1}{2}(e_{12} - e_{21}) = -\omega_{21}$. Likewise for the other

components. Likewise for the other off axis terms. The diagonal terms of the disorientation matrix are all zero.

6. As measured, the strain tensor is obtained relative to reference axes defined with respect to the imaging screen. These are rotated to the specimen reference axes by a rotation about the sample tilt axis, i.e. the x axis. To relate these to crystal axes and in fact to calculate the 9th tensor component as described above we need to know the crystal orientation. We use here the orientation as measured using standard EBSD software. Although the orientation measured in this way is only accurate to 0.5 degrees the strains and rotations obtained using the cross correlation procedures are at 100 times better precision because they are relative to the values of the reference pattern and do not depend on an absolute knowledge of the pattern centre nor specimen to film distance. The absolute orientation is only needed for rotating the strain tensor from specimen reference axes to crystal axes. It has been shown that a half degree error in the measured orientation affects the result by less than 1 part in 10000. The affect of pattern centre on absolute strain measurement is discussed below.
7. Experiment has shown repeatedly that the sensitivity for detecting image shift between the same selected area in two patterns is 5 parts in 100000. After all the math and accumulation of errors this provides for a strain sensitivity of the order 1 part in 10000, i.e. ten times smaller than the elastic limit in metals.
8. To achieve this precision 20 regions of interest (ROI) are selected and there is some filtering of the patterns, using the fast Fourier transform of the pattern. In practice, the image comparison is done in Fourier space for efficiency reasons.
9. The minimum number of regions of interest needed to extract the tensor is 4. Thus because we use up to 20 ROI we are able to obtain a least squares fit to find the best strain tensor that fits all the measured ROI. The raw data can thus be re-examined to see if there were any anomalies signifying poor correlation. There are several approaches. One is to use the cross correlation procedure itself. It automatically gives a quality of fit figure and poor fit values can be removed. Another is to compare measured and theoretical shifts. After the average strain tensor is determined each individual displacement can be examined to test whether it falls within a statistically acceptable range of what is expected according to a back calculation from the average strain tensor. If an individual region fails the test it can be removed and a new average determined. Thus iterative procedures can be adopted to converge on the final strain tensor.

Example

In the earliest uses of the technique we were measuring strain in semiconductor samples. In these cases the devices were strained over a very small range, about 1 micron. Outside this distance the crystal was completely undeformed so that taking a reference pattern at say 3 microns from the strained devices was perfectly safe. We could safely assume the reference pattern was from a strain free region so that all the measurements made relative to this pattern could be taken as the absolute strain in the sample. The results for strain about a micro hardness indent in silicon shown below can with confidence be absolute values within the stated 2 parts in 10000.



Normal strains, shear strains shear stress and rotations as examples of absolute strain measurement together with one stress map, that for σ_{12} .

The figure shows maps of the normal strain components, shear strains and rotations as measured using the XC EBSD procedure and Crosscourt 3 software. The x and y (left) axis define the lateral dimensions scanned during the collection of EBSD patterns and the vertical (right) axis shows the strain or rotation values. Strain is a dimensionless parameter. The rotations are in radians. The vertical axis in the stress map is in Giga Pascals.

Summary

The full description of strain requires 9 components of a displacement tensor to be determined. Eight of these components can be determined by analysis of measured displacements of small parts of an EBSD pattern recorded from a strained part of a crystal from their corresponding positions in an EBSD pattern recorded from an unstrained region of the crystal. The 9th component is calculated on the assumption that the surface is traction free.

The displacement tensor is converted to a strain tensor by dividing the off-diagonal components of the displacement tensor into symmetric and antisymmetric parts.

The measured sensitivity for displacement is 5 parts in 100000 and for strain measurement 2 parts in 10000.

Accuracy depends on knowledge of the strain state of the area from which the reference pattern was obtained. When this is known to be zero then the accuracy is the same as sensitivity that is 2 parts in 10000.

A fuller analysis of data and accuracy is to be given in a second tutorial which will follow shortly.

Bibliography

The following is a list of papers relevant specifically to the technique of strain measurement using EBSD. I have not included those papers that use EBSD for determination of the dislocation tensor and mapping dislocation densities. Nor have I included papers on pattern centre determination and its effects on absolute strain measurements when there is no readily accessible strain free region in the sample. These will appear in later tutorials. Those listed here are in chronological order. The earlier work prior to 2000 is mainly for historic interest.

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